More Parametric Concepts

Name___________________
Period_____Date_________

I. **Setting up a window:** Generally, think about t values that will provide all possible values of x and y. (Basically think about t as the domain of functions x and y and the ranges of x and y as the x and y values of the window.) Then think about how small the t-step must be to provide a continuous graph. Use your calculator to graph each of the following. Sketch each of the graphs and be sure to state your window and t step.

1. $x = 5 + t$ $\quad y = (x-5)^2 + 2$
   $y = t^2 + 2$

2. $x = \ln(t + 3)$ $\quad x = \ln(5-y)$
   $y = 2 - t$

3. $x = 3\cos^2(t) + 7$ $\quad x = \frac{3}{4}(y + 1)^2 + 7$
   $y = 2\cos(t) - 1$

4. $x = 6\sec(t)$ $\quad (x/6)^2 - (y - 8)^2 = 1$
   $y = \tan(t) + 8$

5. $x = \cos(2t)$ $\quad x = 1 - y^2/2$
   $y = 2\sin(t)$

6. $x = 3t + 1$ $\quad y = 4^{(x-1)/3+5}$
   $y = 4^{t+5}$

7. $x = 2^t + 2^{-t}$ $\quad x^2 - y^2 = 4$

   $y = 2^t - 2^{-t}$

8. $x = \dfrac{2}{t-5}$ $\quad y = \dfrac{5x + 2}{8x + 2}$
   $y = \dfrac{t}{t+3}$

9. $x = 4\cdot\cos(\theta)$ $\quad y = 1 \pm x\sqrt{16 - x^2}/8$
   $y = 1 + \sin(2\theta)$

   (Don't let the θ throw you.)

10. $x = t^3 + 1$ $\quad (x - 1)^2 = (y/3)^3$
    $y = 3t^2$

II. **Eliminating the parameter:** There are many ways to eliminate a parameter. A) Solve for t in one of the equations and substitute that value of t into the other. Then simplify the resulting equation (which should be "t free".) B) Think linear combinations. Multiply one equation (or square, etc.) by whatever it takes to enable you to add (or divide, etc.) the equations so that the t values cancel. C) (Especially with trig equations), think about relationships that create a constant, like $\sin^2 t + \cos^2 t = 1$ and $2^t \cdot 2^{-t} = 1$. Then work with the "t side" of the equation to make those expressions happen. Eliminate the parameter in each of the above equations. You know what the graph should look like, so graph the Cartesian equation to check each new equation.

III. **Writing equations of common graphs**: A) Lines: Writing parametric equations of lines are much like writing vector equations of lines. Consider how the x moves and then consider how y moves. Ex. The parametric equation of the line through (3, 6) and (4, 1) would be x = 3 + t and y = 6 – 5t. B) Circles/Ellipses: Think of expressions that, when squared and added, produce constants. The best example is $\cos^2 t + \sin^2 t = 1$. So, let the cos t be an expression involving x and let sin t be an expression involving y. Ex. Consider $\frac{(x-2)^2}{9} + \frac{(y+4)^2}{25} = 1$. If $\frac{(x-2)^2}{9} = \cos^2 t$, then (x - 2)/3 = cos t and x – 2 = 3cos t, so x = 2 + 3·cos t. Similarly, y = -4 + 5·sin t. C) Parabolas: Let the independent variable be t and change the dependent variable to t in the other equation. Write parametric equations of the following:

1. A line through (4, -2) and (7, 1). *x = 4 + 3t and y = -2 + 3t*
2. A line through (-2, 5) with slope of -4/3. *x = -2 + 3t and y = 5 – 4t*
3. A circle with center (6, 4) and radius 3. *x = 6 + 3cos t and y = 4 + 3sin t*
4. A circle with diameter that has endpoints at (-3, 8) and (7, -6). $x = 2 + \sqrt{74}\cos t$
   $y = 1 + \sqrt{74}\sin t$
5. An ellipse with vertices at (-2, 8) and (-2, -4) and one covertex at (1, 2).
   x = -2 + 3cos t and y = 2 + 6sin t
6. An ellipse with center at (2, 4), one focus at (-1, 4) and one vertex at (-3, 4).
   x = 2 + 5cos t y = 4 + 4sin t
7. A hyperbola with vertices at (4, ±6) and asymptotes at y = ±2(x – 4).
   x = 4 + 3tan t y = 6sec t
8. A hyperbola with center (2, -3), a vertex at (2, 0) and a focus at (2, 1).
   $x = 2 + \sqrt{7}\tan t$ $y = -3 + 3\sec t$
9. A parabola with vertex (5 -1) and a directrix at x = -3. $x = t^2 + 5$ $y = 4\sqrt{2}\,t - 1$

IV. **Cool-looking graphs**: Here are just some neat graphs that work well with parametric equations. Don't try to eliminate these parameters – just graph them – sketch them and state the window. You need to be in radians, especially for problems 3 - 6

1. $x = 4\cdot\cos^3 t$
   $y = 4\cdot\sin^3 t$
2. x = 5·cos t – cos (5t)
   y = 5·sin t – sin (5t)
3. x = t + sin t
   y = 1 – cos t
4. x = 2t + sin t
   y = 2 – cos t
5. x = tan t + 5·sin t
   y = 1 + 5·cos t
6. x = cos t + t sin t
   y = sin t - t·cos t