Try these:

I. For each of the following, A) name the type of conic section, B) write its equation in standard form, and C) sketch the graph, showing the focus and directrix.

1. This conic has a focus is at (0, 0), corresponding directrix at y = 3 and eccentricity of 2.

A) It is a hyperbola because the eccentricity is greater than 1.

B)

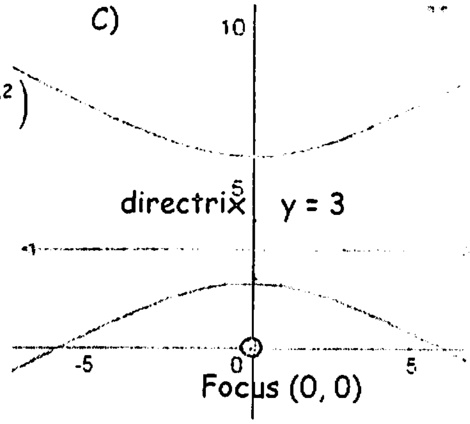
$$\frac{\sqrt{x^2+y^2}}{3-y}=2 \Rightarrow \sqrt{x^2+y^2}=2(3-y) \Rightarrow x^2+y^2=4(9-6y+y^2)$$

$$x^2+y^2=36-24y+4y^2 \Rightarrow x^2-3y^2+24y=36 \Rightarrow$$

$$x^2-3(y^2-8y+16)=36-48=-12$$

$$\frac{x^2}{-12}-\frac{3(y-4)^2}{-12}=1 \Rightarrow \frac{(y-4)^2}{4}-\frac{x^2}{12}=1$$

C)




2. This conic has a focus is at (0, 0), corresponding directrix at x = 1 and eccentricity of 0.6 .

A) It is an ellipse because the eccentricity is less than 1.

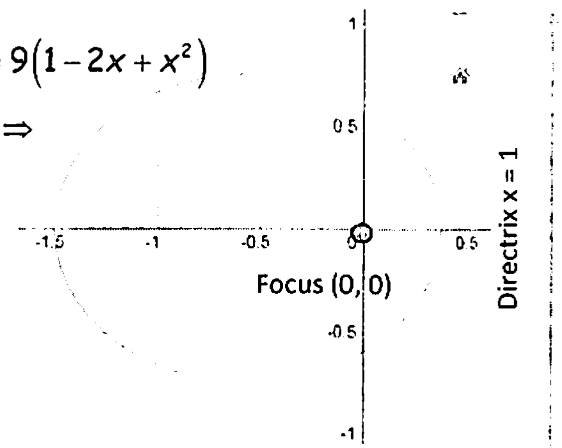
$$\frac{\sqrt{x^2+y^2}}{1-x}=\frac{3}{5} \Rightarrow 5\sqrt{x^2+y^2}=3(1-x) \Rightarrow 25(x^2+y^2)=9(1-2x+x^2)$$

$$25x^2+25y^2=9-18x+9x^2 \Rightarrow 16x^2+25y^2+18x=9 \Rightarrow$$

B) $$16\left(x^2+\frac{9}{8}x+\frac{81}{256}\right)+25(y^2)=9+\frac{81}{16}=\frac{225}{16}$$

$$\frac{16\left(x+\frac{9}{16}\right)^2}{\frac{225}{16}}+\frac{25(y)^2}{\frac{225}{16}}=1 \Rightarrow \frac{\left(x+\frac{9}{16}\right)^2}{\frac{225}{256}}+\frac{y^2}{\frac{9}{16}}=1$$

C)

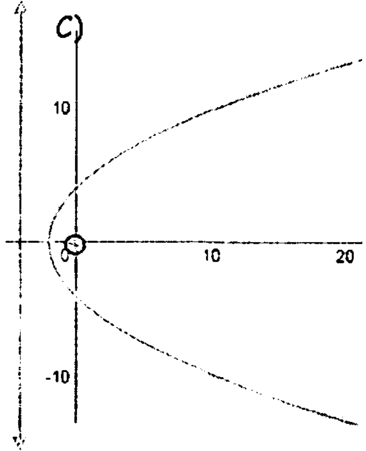



3. This conic has a focus is at (0, 0), corresponding directrix at x = -4 and eccentricity of 1.

A) This figure is a parabola because the eccentricity is 1.

B) Since the vertex is halfway between the focus and the directrix, it must be at (-2, 0). Since the vertex is to the left of the focus, the parabola opens right and since the distance between the vertex and the focus is 2, p = 2. Therefore the equation is: $y^2 = 8(x + 2)$

C)



4. This conic has foci at (-3, 8) and (-3, -4) and eccentricity of 1.2 .

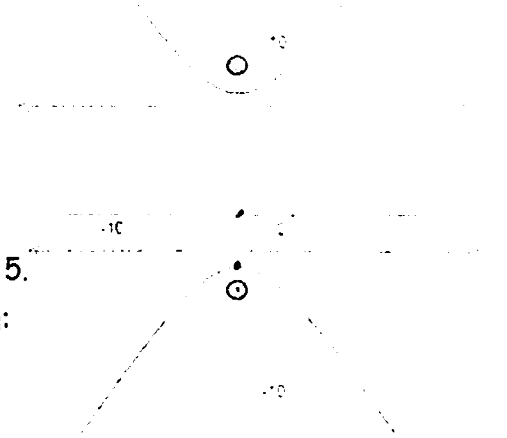
A) Since e = 1.2, this figure is a hyperbola.

B) The center is the midpoint of the foci so it is (-3, 2). Furthermore, since the foci are above and below the center, the hyperbola opens up/down, so the y term is positive. Furthermore, this means c = 6 because that's the distance from the center to a focus. Since e = c/a , 1.2 = 6/a $\rightarrow$ a = 6/1.2 = 5. Since $a^2 + b^2 = c^2$, $25 + b^2 = 36 \rightarrow b^2 = 11$. This makes the equation:

$$\frac{(y-2)^2}{25} - \frac{(x+3)^2}{11} = 1 \quad \text{and } d = \frac{a^2}{c} = \frac{25}{6} \text{ so the asymptotes are}$$

Y = 2 + 25/6 = 37/6 and y = 2 - 25/6 = -13/6

C)



5. This conic has foci at (6, 10) and (6, -14) and eccentricity of 12/13.

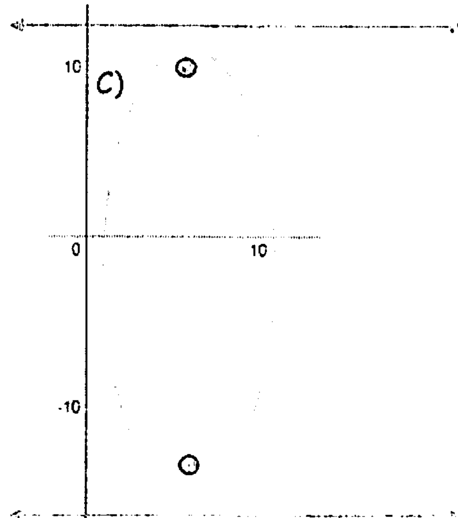
A) Since e = 12/13, the figure is an ellipse.

B) The center is (6, -2) and it opens up/down, so the major axis is parallel to the y axis. c = 12, so a = 13. Since $a^2 = b^2 + c^2$, $b^2 = 169 = 144 = 25$. That makes the equation:

$$\frac{(x-6)^2}{25} + \frac{(y+2)^2}{169} = 1 \quad \text{and } d = \frac{a^2}{c} = \frac{169}{12} \text{ so the asymptotes area}$$

$$Y = -2 \pm \frac{169}{12}$$ or y =145/12 or y = -193/12

C)



6. This conic has a center at (4, 7), one vertex at (21, 7) and eccentricity of 15/17.

A) This is an ellipse.

B) It is longer in the x direction and a = 17 so c = 15.

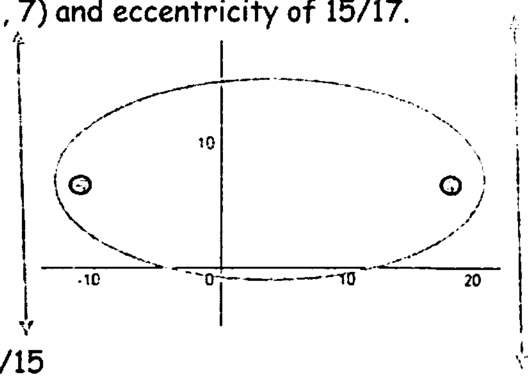
$b^2 = 17^2 - 15^2 = 64$

So the equation is $\frac{(x-4)^2}{289} + \frac{(y-7)^2}{64} = 1$

Foci are at (19, 7) and (-11, 7) and d = 289/15, so

Equations of asymptotes are x = 349/15 and x = -229/15

C)



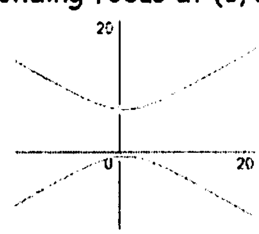
7. This conic has a center at (1, 3), directrix at y = 5 and corresponding focus at (1, 10).

A) Directrix is closer to center than focus, so it's a hyperbola.

B) c = 7 and $d = a^2/c = 2 \rightarrow a^2 = 14$, $a^2 + b^2 = c^2 \rightarrow b^2 = 49 - 14 = 35$

So equation is $\frac{(y-3)^2}{14} - \frac{(x-1)^2}{35} = 1$

C)



8. This conic has a center at (1, 2), directrix at $x = 7$ and corresponding focus at (3, 2).

A) Focus is closer to center than directrix so it's an ellipse.

B) $\frac{(x-1)^2}{12} + \frac{(y-3)^2}{8} = 1$

9. This conic has vertices at (3, 9) and (13, 9) and eccentricity 1/5.

A) Ellipse

B) $\frac{(x-8)^2}{25} + \frac{(y-9)^2}{24} = 1$

10. This conic has vertices at (3, 9) and (13, 9) and eccentricity 5.

A) Hyperbola

B) $\frac{(x-8)^2}{1} - \frac{(y-9)^2}{24} = 1$

11. This conic only has only one vertex at (4, 8) and a focus at (0, 8).

A) Parabola

B) $-16(x - 4) = (y - 8)^2$

12. This conic has an eccentricity of 1/3 and the endpoints of its minor axis are (4, 8) and (0, 8).

A) Ellipse

B) Center is (2, 8), $b = 2$ $c/a = 1/3$, so $a = 3c$. $a^2 = b^2 + c^2 \rightarrow (3c)^2 = 4 + c^2 \rightarrow 8c^2 = 4 \rightarrow c^2 = \frac{1}{2}$

$a^2 = 9(1/2) = 9/2$. So equation is: $\frac{(x-2)^2}{\frac{9}{2}} + \frac{(y-8)^2}{4} = 1$

13. This conic has only one vertex at (3, 2) and a directrix at $x = 0$.

A) Paarabola

B) $12(x - 3) = (y - 2)^2$

14. This conic has an eccentricity of 3/2, a focus at (6, 3) and asymptotes of $y - 3 = \pm 2x$.

A) hyperbola

B) Center (0, 3) and $c = 6$ and since $c/a = 3/2 = 6/a \rightarrow a = 4$ and $a^2 = 16$. $16 + b^2 = 36 \rightarrow b^2 = 20$.

So the equation is: $\frac{(x)^2}{16} - \frac{(y-3)^2}{20} = 1$