Algebra Review Task
Accelerated Math III

Name KEY
Period_____ Date_____

I. Order of Operations with Functions

Evaluate these without a calculator and then check your work using one.

1. If $f(x) = 6x^2 - 5x + 10$, evaluate $f(-2)$. 44
2. If $f(x) = (x - 4)(x + 5)(4x + 3)$, evaluate $f(-2)$. 90
3. If $f(x) = 6 \cdot 7^{(x-1)/2} - 9$, evaluate $f(5)$. 285
4. If $f(x) = 2 - 3\log_9(x + 5)$, evaluate $f(-4\frac{2}{3})$. 7/2
5. If $f(x) = \frac{2x-3}{x+5}$, evaluate $f\left(\frac{-3}{4}\right)$. $-18/17$
6. If $f(x) = 5\sin(6(x - 70°)) + 4$, evaluate $f(100°)$. 4

How did you know to use degrees in this problem?

degree symbol.

7. If $f(x) = 2\sec x - 7$, evaluate $f(\pi/6)$. $\frac{4}{\sqrt{3}} - 7$ or $\frac{4\sqrt{3}}{3} - 7$ or $\frac{4\sqrt{3} - 21}{3}$

How did you know to use radians in this problem?

No degree symbol (unitless)

9. If $f(x) = 8\text{Arcsin}(x + 3) - 5$, evaluate $f(-3.5)$. $-\frac{4\pi}{3} - 5$

Does it matter if you are in radians or degrees? YES Which should be used and why?

RADIANS SINCE "5" IS UNITLESS.

II. So, solve each of the following algebraically without using a calculator.
Check your work using one.

1. $3x + 4 = 15$ $x = 11/3$

How are #1 and #2 different?

order of operations.

2. $3(x + 4) = 15$ $x = 1$

Can #2 be solved another way?

graphing.

3. $3\log_6 x + 9 = 15$ $x = 36$

4. $3\log_6(x + 9) = 15$ $x = 7767$ $3 - \frac{\pi}{3}$

How are #3 and #4 different?

order of op.

5. $3\cos x + \frac{\pi}{3} = 3$ (if $0 \le x < 2\pi$) $\cos x = (1 - \pi/9)$

$x = .862$ and $x = 5.421$

Why do you need a calculator?

Not nice ref. angles.

6. $3\cos\left(x + \frac{\pi}{3}\right) = 3$ (if $0 \le x < 2\pi$)

$\cos^{-1}(1) - \pi/3 = 5\pi/3$

How is this different from #5?

order of op.

How are all the above problems alike?

changing order of operation using ( )

7. If f(x) is a function, solve $a \cdot f(x) + c = y$ for x. $f(x) = \frac{y-c}{a}$ $x = f^{-1}\left(\frac{y-c}{a}\right)$

8. If f(x) is a function, solve $a \cdot f(x+c) = y$ for x. $f(x+c) = \frac{y}{a}$ $x = f^{-1}\left(\frac{y}{a}\right) - c$

III. Keep solving these without technology. Check using it, though.

1. $x^2 - 9x = 0$ $x(x-9)$ $x=0$ $x=9$

1'. $4x^2 - x = 0$ $x(4x-1)$ $x=0$ $x=1/4$

How is this different from $x^2 = 9x$?? SAME

But what happens if we divide by x?? LOSE $x=0$ AS AN ANSWER.

2. $x^2 - 9 = 0$ $x = \pm 3$

2'. $4x^2 - 1 = 0$ $x = \pm 1/2$

How is this different from $x^2 = 9$?? SAME

3. $2\sin^2 x - \sin x = 0$ (if $0 \le x < 2\pi$) $\sin x(2\sin x - 1) = 0$ $x = 0, \pi, \pi/6, 5\pi/6$

How is this different from $2\sin^2 x = \sin x$? SAME

4. $2\sin^2 x - 1 = 0$ (if $0 \le x < 2\pi$) $\sin^2 x = 1/2$ $\sin x = \pm \frac{1}{\sqrt{2}}$ $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

How is this different from $2\sin^2 x = 1$? SAME

5. $x^2 - 5x + 4 = 0$ $(x-4)(x-1) = 0$ $x = 4, x = 1$

How is this different from $x^2 + 4 = 5x$? NOT DIFFERENT

6. $x^2 - 5x + 4 = -2$ $x^2 - 5x + 6 = (x-6)(x+1) = 0$ $x = 6, -1$

So, how must we solve quadratics? FACTOR, COMPLETE SQUARE, QUAD FORMULA

7. $\left(\frac{2x-5}{3}\right)^2 - 5\left(\frac{2x-5}{3}\right) + 4 = 0$ $\frac{2x-5}{3} = 4$ $x = 17/2$ $\frac{2x-5}{3} = 1$ $x = 4$

How can we identify quadratics? $(\ )^2 + (\ ) + k$

8. $\csc^2 x - \csc x - 2 = 0$ (if $0 \le x < 2\pi$) $\csc x = a \Rightarrow a^2 - a - 2 = 0$ $(a-2)(a+1) = 0$ $\csc x = 2$ $\csc x = -1 \Rightarrow \sin^{-1}(1/2)$ $\sin^{-1}(-1)$

Can you check by graphing? YES

9. $2\cos^2 x - 3\cos x + 1 = 0$ (if $0 \le x < 2\pi$) $(2x-1)(x+1)$ $\cos x = 1/2$ $\cos x = 1$ $x = \pi/6, 5\pi/6, \pi$ $x = \pi/3, 5\pi/3, 0$

10. $x^2 + x + 10 = 7x$ $x^2 - 6x + 10 = 0$ $\frac{6 \pm \sqrt{36 - 4(10)}}{2} = \frac{6 \pm 2i}{2} = 3 \pm i$

11. $4\tan^2 x - 2\tan x = 5$ (if $0 \le x < 2\pi$) $4a^2 - 2a - 5$ $a = \frac{2 \pm \sqrt{104}}{8} = \tan x$ $x = 56.74^\circ = .99$, $236.74^\circ = 4.13$, $x = 314.3^\circ = 5.49$, $134.3^\circ = 2.34$

What do we do when quadratics don't factor? QUADRATIC FORMULA

12. $(x-5)(2x-3) = 0$ $x = 5$ $x = 3/2$

Is this the same as $x - 5 = 0$ or $2x - 3 = 0$? YES

13. $(x-5)(2x-3) = 6$ $2x^2 - 13x + 15 = 6$ $2x^2 - 13x + 9 = 0$ $x = \frac{13 \pm \sqrt{97}}{4} = .788 \& 5.71$

Is this the same as $x - 5 = 6$ or $2x - 3 = 6$? NO

Is this the same as $x - 5 = 2$ or $2x - 3 = 3$? NO

Why doesn't factoring work here??? NOT = ZERO

14. $(3\sin x - 2)(\sin x - 1) = 2$ (if $0 \le x < 2\pi$) $3\sin^2 x - 5\sin x + 2 = 2$ $3\sin^2 x - 5\sin x = 0$ $\sin x(3\sin x + 5) = 0$ $x = 0, \pi$

IV. Keep solving each of the following algebraically without using a calculator.
Check your work using one.

1. $\frac{1}{x} = 7$ $x = \frac{1}{7}$ What property is helpful? multiplication prop of equality.
2. $\frac{1}{x-4} = 7$ $7(x-4) = 1$ $7x = 29$ $x = 29/7$
3. $\frac{13}{x-4} = 7$ $x = 41/7$ Is $\frac{13}{x-4} = 13 \cdot \frac{1}{x-4}$? YES.
4. $\frac{13}{2(x-4)} = 7$ $13 = 14x - 56$ $x = 69/14$
5. $\frac{13x}{x-4} = 7$ $13x = 7x - 28$ $x = \frac{-28}{6} = -14/3$
6. $\frac{3}{\sin x} = 6$ (if $0 \le x < 2\pi$) $\sin^{-1}(1/2) = \pi/6, 5\pi/6$
7. $3\sec x = 6$ (if $0 \le x < 2\pi$) Solve this at least 3 ways!
   ① $\sec x = 2$ $\cos x = \frac{1}{2}$ $x = \pi/3, 5\pi/3$ ② Graph. ③ $\sec x = 2$ @ $5\pi/3, \pi/3$.
8. If $f(x) = \frac{2x-5}{x+6}$, write an equation for $f^{-1}(x)$. $x = \frac{2y-5}{y+6}$

$xy + 6x = 2y - 5$
$6x + 5 = 2y + xy$
$6x + 5 = y(x+2)$
$\frac{6x+5}{x+2} = y$

V. Common misconceptions...

1. Is $2 + 3 = 5$? YES Is $2^2 + 3^2 = 5^2$? NO
   Is $(2 + 3)^2 = 2^2 + 2(2 \cdot 3) + 3^2 = 5^2$? YES.
2. Is $3^2 + 4^2 = 5^2$? YES. Is $3 + 4 = 5$? NO
   Is $\sqrt{3^2 + 4^2} = \sqrt{5^2}$? YES. ← I.D.
3. When is $\cos^2 x + \sin^2 x = 1$? ALWAYS Find an x so that $\cos x + \sin x \neq 1$. any angle except $0 + 360°k$ or $90° + 360°k$
   So, when is $\cos x + \sin x = 1$? Find at least 2 ways to solve this.

   Write an expression for cos x in terms of sin x. $\cos x = \sin(90° - x)$
4. Is $\tan^2 \theta + 1 = \sec^2 \theta$? YES Is $\sec \theta = \tan \theta + 1$? NO
5. Write a sentence about squaring or taking roots of an equation.
   MUST SQUARE / SQUARE ROOT ENTIRE SIDE
6. Why is it difficult solve $x^2 + 4y = 7$? 2 VARIABLES,
7. But how do we overcome that to solve $\cos^2 x + \sin x = 1$?
   TRIG IDENTITIES.
8. Solve $\cos^2 x + \sin x = 1$ (if $0 \le x < 2\pi$)

$1 - \sin^2 x + \sin x = 1$ $\sin x(-\sin x + 1) = 0$
$-\sin^2 x + \sin x = 0$ $x = 0, \pi$ $\pi/2,$

9. Is $(5 - 2)^2 = 5^2 - 2^2$? NO Is $\cos(30° + 60°) = \cos 30° + \cos 60°$? NO

Is $\sqrt{3+6} = \sqrt{3} + \sqrt{6}$? NO Is $\log_2(2 + 8) = \log_2 2 + \log_2 8$? NO.

10. Write a sentence about distributing a function. ONLY WITH MULTIPLICATION and DIVISION

11. How does the graph of y = 4cos 3x differ from the graph of y = 4cos x?

PERIOD IS 1/3 AS LONG

How does the graph of y = 4cos 3x differ from the graph of y = cos 3x ?

SAME AS ABOVE

Describe how the intersections of y1 = 4cos x and y2 = -2 differs from the intersections of y1 = 4cos 3x and y2 = -2.

1/3 THE VALUES.

When solving 4cos 3x = -2 for $0 \le x < 2\pi$, what must be done to consider all possible solutions? CONSIDER VALUES FROM 2π to 6π SINCE THEY WILL BE $0 \le x \le 2\pi$ WHEN DIVIDED BY 3.

Do that to solve 4cos 3x = -2 algebraically.

$\cos 3x = -1/2$

$3x = 120° + 360°k$ $3x = 240° + 360°k$

$x = 40° + 120°k$ $x = 80° + 120°k$

$x = 40°, 160°, 280°, 80° + 200°, 320°$

VI. Speaking of Fractions...

1. Is $\frac{5+2}{5+7} = \frac{1+2}{1+7}$? NO Is $\frac{5+2}{5+7} = \frac{2}{7}$? NO Is $\frac{5}{5+7} = \frac{1}{1+7}$? NO

Is $\frac{5+10}{5+20} = \frac{1+1}{1+2}$? NO Is $\frac{5+10}{5+20} = \frac{1+2}{1+4}$? YES. Is $\frac{5}{5+20} = \frac{1}{1+4}$? YES.

Is $\frac{4+\sqrt{5}}{2} = 2 + \sqrt{5}$? NO Is $\frac{5+2\sin x}{5} = 1 + 2\sin x$? NO Is $\frac{5}{5+10\ln x} = \frac{1}{1+2\ln x}$? YES.

2. Write a statement about reducing fractions.

IN A FRACTION, THE QUANTITIES MUST BE MULTIPLIED BY EACH OTHER, SO MUST FACTOR FIRST

3. Is $\frac{5+4}{3} = \frac{5}{3} + \frac{4}{3}$? YES. Is $\frac{3}{5+4} = \frac{3}{5} + \frac{3}{4}$? NO What is the difference?

NUMBER OF TERMS IN DENOMINATOR

4. So is $\frac{\sin x + \cos x}{\sin x} = 1 + \cot x$? YES Is $\frac{\sin x}{\sin x + \cos x} = 1 + \tan x$? NO