

Consider the quadratic function: $f(x) = 2x^2 - 5x + 7$

Let's make a table of $f(x)$ for $x \in \{1, 2, 3, 4, 5\}$.

x	1	2	3	4	5
f(x)	4	5	10	19	32

Now let's find the **difference** between each successive term - successive differences

x	1	2	3	4	5
f(x)	4	5	10	19	32
Successive Differences	1	5	9	13	

What pattern do you see? **The differences differ by a constant 4.**

Now let's find the difference between the differences, and then the differences between those differences:

x	1	2	3	4	5
f(x)	4	5	10	19	32
Successive Differences	1	5	9	13	
Second Differences		4	4	4	

If the first differences are **constant**, then the function is linear.

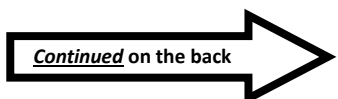
But when the function is **quadratic**, what is true? Second differences are constant.

What would you expect to be true if the third differences are constant?

The function should be cubic (3rd degree).

Check out your hypothesis with an example.

My $f(x) =$ <u>$x^3 - 5x^2 + 6x - 2$</u>	x	1	2	3	4	5
	f(x)	0	-2	-2	6	28
	Successive Differences	-2	0	8	22	
	Second Differences		2	8	14	
	Third Differences			6	6	



Let's look at this pattern in general for $f(x) = ax^2 + bx + c$. Complete the table...

x	1	2	3	4	5
$f(x)$	$a+b+c$	$4a+2b+c$	$9a+3b+c$	$16a+4b+c$	$25a+5b+c$
Successive Differences	$3a+b$	$5a+b$	$7a+b$	$9a+b$	
Second Differences		$2a$	$2a$	$2a$	
Third Differences		0	0		

This means if our function (or sequence) is **quadratic**, we can use this information to write its equation or general term. Let's try it with the following sequence: $\{-11, -12, -9, -2, 9\}$

n	1	2	3	4	5
a_n	-11	-12	-9	-2	9
Successive Differences	-1	3	7	11	
Second Differences		4	4	4	
Third Differences		0	0		

So $2a = \underline{4}$, meaning $a = \underline{2}$. And $3a + b = \underline{-1}$, so $b = \underline{-7}$.

And since $a + b + c = -11$, $c = \underline{-6}$.

This means the explicit form for our sequence is: $a_n = \underline{2n^2 - 7n - 6}$.

Now, try to write equations for these sequences:

1.) 4, 10, 18, 28, 40, ...

$$a_n = n^2 + 3n$$

2.) 7, 8, 11, 16, 23, ...

$$a_n = n^2 - 2n + 8$$

3.) -3, -11, -25, -45, -71, ...

$$a_n = n^2 - 2n + 8$$

4.) -2, 4, 10, 16, 22, ...

$$a_n = 6n - 8$$

5.) -7, -3, 3, 11, 21, ...

$$a_n = n^2 + n - 9$$

6.) 12, 15, 16, 15, 12, ...

$$a_n = -n^2 + 6n + 7$$