Acc.Precalculus Name **Solutions** .

A Look at Successive Differences Period ______Date_____________

Consider the quadratic function: $f(x) = 2x^2 - 5x + 7$

Let's make a table of $f(x)$ for $x \in \{1, 2, 3, 4, 5\}$.

x	1	2	3	4	5
$f(x)$	4	5	10	19	32

Now let's find the **difference** between each successive term – successive differences

x	1	2	3	4	5
$f(x)$	4	5	10	19	32
Successive Differences	1	5	9	13	

What pattern do you see? *The differences differ by a constant 4.*

Now let's find the difference between the differences, and then the differences between those differences:

x	1	2	3	4	5
$f(x)$	4	5	10	19	32
Successive Differences	1	5	9	13	
Second Differences	4	4	4		

If the first differences are **constant**, then the function is ______*linear*______.

But when the function is **quadratic**, what is true? *Second differences are constant.*

What would you expect to be true if the third differences are constant?

The function should be cubic (3rd degree).

Check out your hypothesis with an example.

My $f(x) =$ x^3-5x^2+6x-2

x	1	2	3	4	5
$f(x)$	0	-2	-2	6	28
Successive Differences	-2	0	8	22	
Second Differences	2	8	14		
Third Differences	6	6			

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Let's look at this pattern in general for $f(x) = ax^2 + bx + c$. Complete the table...

x	1	2	3	4	5
$f(x)$	a+b+c	4a+2b+c	9a+3b+c	16a+4b+c	25a+5b+c
Successive Differences	3a+b	5a+b	7a+b	9a+b	
Second Differences	2a	2a	2a		
Third Differences	0	0			

This means if our function (or sequence) is **quadratic**, we can use this information to write its equation or general term. Let's try it with the following sequence: {-11 , -12 , -9 , -2 , 9}

n	1	2	3	4	5
a_n	-11	-12	-9	-2	9
Successive Differences	-1	3	7	11	
Second Differences	4	4	4		
Third Differences	0	0			

So $2a =$ 4, meaning $a =$ 2. And $3a + b =$ -1, so $b =$ -7.

And since $a + b + c = -11$, $c =$ -6.

This means the explicit form for our sequence is: $a_n = 2n^2 - 7n - 6$.

Now, try to write equations for these sequences:

1.) 4 , 10 , 18 , 28 , 40 , ... $a_n = n^2 + 3n$

2.) 7 , 8 , 11 , 16 , 23 , ... $a_n = n^2 - 2n + 8$

3.) -3 , -11 , -25 , -45 , -71 , ... $a_n = n^2 - 2n + 8$

4.) -2 , 4 , 10 , 16 , 22 , ... $a_n = 6n - 8$

5.) -7 , -3 , 3 , 11 , 21 , ... $a_n = n^2 + n - 9$

6.) 12 , 15 , 16 , 15 , 12 , ... $a_n = -n^2 + 6n + 7$